

A Novel Procedure to aid Breast Cancer Biopsy using Wavelet based Active Contour model for Mammographic Lesions

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Abstract: *This research proposes a novel procedure to aid surgical biopsy using wavelet based active contour model. Surgical Biopsy is standard tool for determination of carcinoma. This research work combines active contour model segmentation with surgical biopsy to propose a procedure that is minimally invasive and accurate. Mammography is the primary imaging technique for the detection and diagnosis of breast lesions. A new segmentation technique using Wavelet based active contour segmentation algorithm has been proposed and tested for segmenting the mammogram lesions in this paper. This new technique incorporates wavelet refinement for better fitting. Experimental studies establish that the proposed segmentation technique gives better segmentation results in detecting the mammographic lesion in comparison to few other existing algorithms. These results can then be used to aid surgical biopsy procedures for accurate and minimally invasive biopsy.*

Index Terms: *Biopsy, mammogram, active contour modeling, wavelets, image segmentation.*

I. INTRODUCTION

Breast cancer is one of the significant health concerns that has started to claim prominence in Medical and Allied Research due to its high prevalence and detection rates since the last few decades. It has been reported that One in eight women in the United States will develop breast cancer during her lifetime [1]. Prevention of Breast Cancer is a hitherto unreliable solution though possible. This necessitates that early detection is mandatory to reduce or prevent loss of life due to breast cancer.

Mammography is the primary imaging technique for the detection and diagnosis of breast lesions. However, due to various fatigue and human factors, the miss rate has been high.

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It has been observed that radiologists miss about 10% of all cancerous lesions [2]. Also the overall percentage of breast cancer detected per number of breast biopsies performed on the basis of mammographic screening ranges between 10% and 50% [3].

Though clinical testing of lesion detection algorithms is routinely done, yet there is a constant endeavor to achieve high performance. Also, algorithms in each category represent different image-processing methodologies. Hence this research work proposes a Novel segmentation technique using wavelet based active contour model for detection of mammographic lesions. Comparison with few other existing algorithms establishes that active contour model performs better in detecting mammographic lesions. In this paper, two automatic mammographic lesion detection algorithms – Fuzzy C-means Clustering, Otsu thresholding, are compared with our Active Contour model based algorithm.

II. METHODS

A. Global methods

Texture analysis [4] allows us to segment the image and classify the segmented regions. These regions can produce parameters for a forthcoming method. The segmentation (or region of interest localization) of the mammograms consists of three parts: preprocessing, “coarse” segmentation and “fine” segmentation. Preprocessing is used to extract the true breast region from the image using thresholding [5] and median filtering.

Further algorithms are working only on the extracted region. The second step (“coarse” segmentation) calculates various texture parameters (co-occurrence matrix [6], gray level run length [4], gray level differences [4] and histogram features [4] in a particular window to create the feature vector. The feature vector is passed to a set of decision trees [7], which will classify the actual image segment (in which the feature vector is calculated). The decision trees are automatically generated from a set of training images.

These training images contain a mass (or lesion) and some surrounding background tissue. The image is segmented using all the decision trees. The output is a vote board where each image segment can have a vote value between zero and the number of the classifying trees. This vote board is post processed to create a binary mask that will cover regions of interests (suspected locations of lesions).

It is treated as an image and adaptive filtering is applied to locate the most suspicious regions. The “fine” segmentation step uses a multiresolution Markov random field [8, 9] to improve the preliminary segmentation provided by the “coarse” segmentation.

B. Local methods

Another segmentation is performed on the patches using a combination of dual binarization [8], Bèzier histograms [5, 10] and a modification of the radial gradient index method [5] to obtain a black-and-white mask. After getting the mask of the lesion candidate several parameters are calculated. Some of them refer to the shape of the object (e.g. moments [5], a special symmetry measure based on PCA [5], compactness [11]; others refer to the texture of the object and its surroundings (e.g. average brightness of the masked area, average brightness of the background, the quotient of these two numbers; and the average variance of the masked and unmasked regions).

Based on these parameters, it can be decided whether the patch contains a true lesion or not. Currently human experts evaluate the parameters, but the development of an automatic clustering module is in progress.

In this paper, two automatic mammographic lesion detection algorithms – Fuzzy C-means Clustering, Otsu thresholding, are compared with our Active Contour model based algorithm.

C. Fuzzy C-means Clustering

Fuzzy C-means Clustering (FCM), is also known as Fuzzy ISODATA employs fuzzy partitioning such that a data point can belong to all groups with different membership grades between 0 and 1. FCM is an iterative algorithm. The aim of FCM is to find cluster centers (centroids) that minimize a dissimilarity function.

To accommodate the introduction of fuzzy partitioning, the membership matrix (U) is randomly initialized according to Equation 1.

$$\sum_{i=1}^c u_{ij} = 1, \forall j = 1, \dots, n \quad (1)$$

The dissimilarity function which is used in FCM is given Equation 2.2

$$J(U, c_1, c_2, \dots, c_c) = \sum_{i=1}^c J_i = \sum_{i=1}^c \sum_{j=1}^n u_{ij}^m d_{ij}^2 \quad (2)$$

u_{ij} is between 0 and 1;

c_i is the centroid of cluster i ;

d_{ij} is the Euclidian distance between i_{th} centroid (c_i) and j_{th} data point;

$m \in [1, \infty]$ is a weighting exponent.

To reach a minimum of dissimilarity function there are two conditions. These are given in Equation 3 and Equation 4

$$c_i = \frac{\sum_{j=1}^n u_{ij}^m x_j}{\sum_{j=1}^n u_{ij}^m} \quad (3)$$

$$u_{ij} = \frac{1}{\sum_{k=1}^c \left(\frac{d_{ij}}{d_{kj}} \right)^{2/(m-1)}} \quad (4)$$

By iteratively updating the cluster centers and the membership grades for each data point, FCM iteratively moves the cluster centers to the "right" location within a data set.

FCM does not ensure that it converges to an optimal solution. Because of cluster centers (centroids) are initialize using U that randomly initialized (Equation 3).

D. Otsu Thresholding

Otsu's method [7] chooses the optimal thresholds by maximizing the between-class variance with an exhaustive search.

III. ACTIVE CONTOUR MODEL

Active contours “snakes” can be used to segment objects automatically. The basic idea is the evolution of a curve, or curves subject to constraints from the input data. The curve should evolve until its boundary segments the object of interest. This framework has been used successfully by Kass et al. [12] to extract boundaries and edges. One potential problem with this approach is that the topology of the region to be segmented must be known in advance.

The propagating curve is modeled as a specific level set of a higher dimensional surface. It is common practice to model this surface as a function of time. So as time progresses, the surface can change to take on the desired shape.

A. Mathematical Formulation of Level Sets

Let Ω be a bounded open subset of R^2 , with $d\Omega$ as its boundary. Then a two dimensional image u_0 can be

defined as $u_0 : \Omega \rightarrow \mathbb{R}$. In this case Ω is just a fixed rectangular grid. Now consider the evolving curve C in Ω , as the boundary of an open subset ω of Ω . In other words $\omega \subseteq C$, and C is the boundary of ω ($C = \partial\omega$).

The main idea is to embed this propagating curve as the zero level set of a higher dimensional function ϕ . We define the function as follows:

$$\phi(x, y, t=0) = \pm d$$

where d is the distance from (x,y) to $\partial\omega$ at $t = 0$, and the plus (minus) sign is chosen if the point (x,y) is outside (inside) the subset ω .

Now, the goal is to produce an equation for the evolution of the curve. Evolving the curve in the direction of its normal amounts to solving the partial differential equation [13]:

$$\frac{\partial \phi}{\partial t} = F |\nabla \phi|, \phi(x, y, 0) = \phi_0(x, y)$$

where the $\{(x,y), \phi_0(x,y) = 0\}$ defines the initial contour, and F is the speed of propagation. For certain forms of the speed function F , this reduces to a standard Hamilton-Jacobi equation. There are several major advantages to this formulation. The first is that $\phi(x,y,t)$ always remains a function as long as F is smooth. As the surface ϕ evolves, the curve C may break, merge, and change topology.

Another advantage is that geometric properties of the curve are easily determined from a particular level set of the surface ϕ . For example, the normal vector for any point on the curve C is given by:

$$\vec{n} = \nabla \phi$$

and the curvature K is obtained from the divergence of the gradient of the unit normal vector to the front:

$$K = \text{div} \left(\frac{\nabla \phi}{|\nabla \phi|} \right) = \frac{\phi_{xx}\phi_y^2 - 2\phi_x\phi_y\phi_{xy} + \phi_{yy}\phi_x^2}{(\phi_x^2 + \phi_y^2)^{3/2}}$$

Finally, another advantage is that we are able to evolve curves in dimensions higher than two. The above formulae can be easily extended to deal with higher dimensions. This is useful in propagating a curve to segment volume data.

B. Active Contours without Edges

Recall that the curve C can be viewed as the boundary of an open subset ω of Ω (i.e. $C = \partial\omega$). Denote the region ω by $\text{inside}(C)$ and the region $\Omega \setminus \omega$ by $\text{outside}(C)$. Now rather than basing the model on an edge-stopping function, we will halt the evolution of the curve with a energy minimization approach.

Consider a simple case where the image u_0 is formed by two regions of piecewise constant intensity. Denote

the intensity values by $u_{0,0}$ and $u_{0,1}$. Furthermore, assume that the object to be detected has a region whose boundary is C_0 and intensity $u_{0,1}$. Then inside (C_0), the intensity of u_0 is approximately $u_{1,0}$, whereas outside(C_0) the intensity of u_0 is approximately $u_{0,0}$. Then consider the fitting term:

$$F_1(C) + F_2(C) = \int_{\text{inside}(C)} |u_0(x, y) - c_1|^2 dx dy + \int_{\text{outside}(C)} |u_0(x, y) - c_2|^2 dx dy$$

where C is a curve, and the constants c_1, c_2 are the averages of u_0 inside and outside of C respectively. Consider Figure 4, If the curve C is outside the object, then $F_1(C) > 0$, $F_2(C) \approx 0$. If the curve is inside the object, then $F_1(C) \approx 0$, $F_2(C) > 0$. If the curve is both inside and outside the object, then $F_1(C) > 0$; $F_2(C) > 0$. However, if the curve C is exactly on our object boundary C_0 , then $F_1(C) \approx 0$, $F_2(C) \approx 0$, and our fitting term is minimized.

We also consider adding some regularization terms as in the Mumford-Shah segmentation model [14]. Therefore we will also try to minimize the length of the curve and the area of the region inside the curve. So we introduce the energy function E :

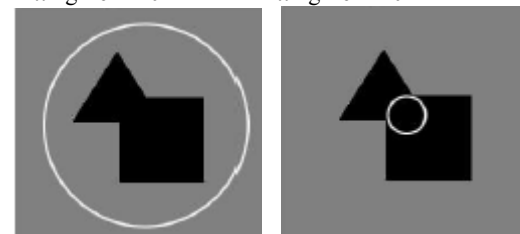
$$E(C, c_1, c_2) = \mu \text{Length}(C) + \nu \text{Area}(\text{inside}(C)) + \lambda_1 \int_{\text{inside}(C)} |u_0(x, y) - c_1|^2 dx dy + \lambda_2 \int_{\text{outside}(C)} |u_0(x, y) - c_2|^2 dx dy$$

where $\mu \geq 0$, $\nu \geq 0$, $\lambda_1 > 0$, $\lambda_2 > 0$ are fixed parameters. So our goal is to find C , c_1 , c_2 such that $E(C, c_1, c_2)$ is minimized.

Mathematically, we want to solve:

$$\sum_{i=1}^c u_{ij} = 1, \forall j = 1, \dots, n$$

$F_1(C) > 0, F_2(C) \approx 0$ $F_1(C) \approx 0, F_2(C) > 0$
Fitting Term > 0 Fitting Term > 0



$F_1(C) > 0, F_2(C) > 0$
Fitting Term > 0

$F_1(C) \approx 0, F_2(C) \approx 0$
Fitting Term ≈ 0

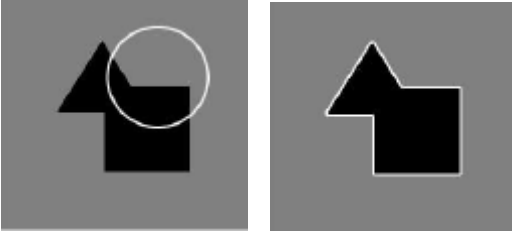


Figure 1: All possible cases in position of the curve.

This problem can be formulated using level sets as follows. The evolving curve C can be represented by the zero level set of the signed distance function ϕ as in (1). So we replace the unknown variable C by ϕ . Now consider the Heaviside function H , and the Dirac measure δ :

$$H(z) = \begin{cases} 1 & \text{if } z \geq 0 \\ 0 & \text{if } z < 0 \end{cases}, \quad \partial(z) = \frac{d}{dz} H(z)$$

We can rewrite the length of $\phi = 0$ and the area of the region inside($\phi = 0$) using these functions. The Heaviside function is positive inside our curve and zero elsewhere, so the area of the region is just the integral of the Heaviside function of ϕ . The gradient of the Heaviside function defines our curve, so integrating over this region gives the length of the curve.

Mathematically:

$$\text{Area}(\phi = 0) = \int_{\Omega} H(\phi(x, y)) dx dy$$

$$\begin{aligned} \text{Length}(\phi = 0) &= \int_{\Omega} |\nabla H(\phi(x, y))| dx dy \\ &= \int_{\Omega} \partial(\phi(x, y)) |\nabla \phi(x, y)| dx dy \end{aligned}$$

Similarly, we can rewrite the previous energy equations so that they are defined over the entire domain rather than separated into inside(C) = $\phi > 0$ and outside(C) = $\phi < 0$:

$$\begin{aligned} \int_{\phi > 0} |u_0(x, y) - c_1|^2 dx dy &= \int_{\Omega} |u_0(x, y) - c_1|^2 H(\phi(x, y)) dx dy \\ \int_{\phi < 0} |u_0(x, y) - c_2|^2 dx dy &= \int_{\Omega} |u_0(x, y) - c_2|^2 (1 - H(\phi(x, y))) dx dy \end{aligned}$$

Therefore our energy function $E(C, c_1, \phi)$ can be written as:

$$\begin{aligned} E(C, c_1, c_2) &= \mu \int_{\Omega} \partial(\phi(x, y)) |\nabla \phi(x, y)| dx dy \\ &+ \nu \int_{\Omega} H(\phi(x, y)) dx dy \\ &+ \lambda_1 \int_{\Omega} |u_0(x, y) - c_1|^2 H(\phi(x, y)) dx dy \\ &+ \lambda_2 \int_{\Omega} |u_0(x, y) - c_2|^2 (1 - H(\phi(x, y))) dx dy \end{aligned}$$

The constants c_1, c_2 are the averages of u_0 in $\phi \geq 0$ and $\phi < 0$ respectively.

So they are easily computed as:

$$c_1(\phi) = \frac{\int_{\Omega} u_0(x, y) H(\phi(x, y)) dx dy}{\int_{\Omega} H(\phi(x, y)) dx dy}$$

and

$$c_2(\phi) = \frac{\int_{\Omega} u_0(x, y) (1 - H(\phi(x, y))) dx dy}{\int_{\Omega} (1 - H(\phi(x, y))) dx dy}$$

Now we can deduce the Euler-Lagrange partial differential equation. We parameterize the descent direction by $t \geq 0$, so the equation $\phi(x, y, t)$ is:

$$\frac{\partial \phi}{\partial t} = \partial \phi \left[\mu \text{div} \left(\frac{\nabla \phi}{|\nabla \phi|} \right) - \nu - \lambda_1 (u_0 - c_1)^2 + \lambda_2 (u_0 - c_2)^2 \right] = 0$$

In order to solve this partial differential equation, we first need to regularize $H(z)$ and $\delta(z)$.

$$H_{\epsilon}(z) = \frac{1}{2} + \frac{1}{\pi} \arctan \left(\frac{z}{\epsilon} \right)$$

implying that $\delta(z)$ regularizes to:

$$\partial(z) = \frac{1}{\pi} \cdot \frac{\epsilon}{\epsilon^2 + z^2}$$

It is easy to see that as $\epsilon \rightarrow 0$, $H_{\epsilon}(z)$ converges to $H(z)$ and $\delta_{\epsilon}(z)$ converges to $\delta(z)$. The authors mention that with these regularizations, the algorithm has the tendency to compute a global minimizer.

After discretization and linearization, it becomes:

$$\begin{aligned} \frac{\phi_{i,j}^{n+1} - \phi_{i,j}^n}{\Delta t} &= \partial_{\epsilon}(\phi_{i,j}^n) \left[\frac{\mu}{h^2} \Delta_{-}^x - \right. \\ &\quad \cdot \left(\frac{\Delta_{+}^x \phi_{i,j}^{n+1}}{\sqrt{(\Delta_{+}^x \phi_{i,j}^n)^2 / h^2 + (\phi_{i,j+1}^n - \phi_{i,j-1}^n)^2 / (2h)^2}} \right) \\ &\quad + \frac{\mu}{h^2} \Delta_{-}^x - \\ &\quad \cdot \left(\frac{\Delta_{+}^y \phi_{i,j}^{n+1}}{\sqrt{(\Delta_{+}^y \phi_{i,j}^n)^2 / h^2 + (\phi_{i+1,j}^n - \phi_{i-1,j}^n)^2 / (2h)^2}} \right) \\ &\quad \left. - \nu - \lambda_1 (u_{0,i,j} - c_1(\phi^n))^2 + \lambda_2 (u_{0,i,j} - c_2(\phi^n))^2 \right] \end{aligned}$$

where the forward differences of $\phi_{i,j}^n$ are calculated.

This linear system also depends on the forward differences of $\phi_{i,j}^{n+1}$, which is an unknown. However these can be solved using the Jacobi method. In

practice, the number of iterations until convergence was found to be small.

C. Active contour modeling with wavelets

The idea of the scale-space continuation method [15] is to calculate the snake in a coarsely smoothed image; then the result at the coarse scale is used as an initial contour on a finer image and so on, until the native image resolution is reached. The original image is filtered through a family of Gaussian filters with different resolutions. Then, a differentiating filter, such as the Sobel filter, is applied to these Gaussian filtered images to produce approximations of the gradients of the Gaussian smoothed image.

The following definition of external energy together with the continuation method in the wavelet domain represents a generalized version of the gradient-based scale-space continuation method. It has been shown [16] that fast implementation of can be achieved when s is an integer power of 2 by filtering alternatively through a low-pass filter (L) and a highpass filter (H). Then, the external energy at scale s is defined as the negative of the modulus of wavelet transform at scale s

$$E_{image}(x, y) = E_{ext}^s(x, y) = -\sqrt{|W_s^1 I(x, y)|^2 + |W_s^2 I(x, y)|^2}$$

We employ this wavelet- based snake model in our experiments on mammogram images. The flow chart of utilized model is shown in Figure 2.

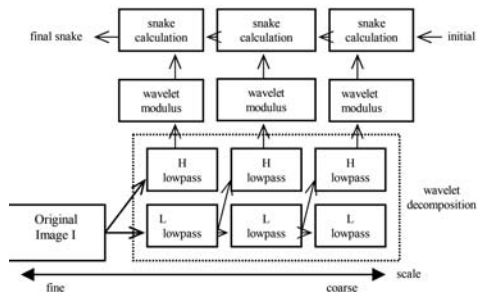


Figure 2. Flow chart of the wavelet-based active contour model

D. Proposed Segmentation Algorithm

The proposed Energy Minimization Algorithm with Jacobi Method is given as:

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Initialize  $\phi^0$  by  $\phi_0$ ,  $n = 0$ 
for fixed number of iterations do
    Compute  $c_1(\phi^n)$  and  $c_2(\phi^n)$ 
    Estimate forward differences of  $\phi^{n+1}$  using Jacobi method
    Compute  $\phi^{n+1}$ 
End
  
```

Using the energy minimization approach, we achieve the desired segmentation in digital mammogram images.

IV. SEGMENTATION RESULTS AND DISCUSSION

A. Segmentation Results

Images from the MIAS (Mammographic Image Analysis Society) database with lesions has been tested.

To indicate the effectiveness of the proposed method, we present a sample image and segmentation results using the proposed method and two other methods namely FCM clustering and Otsu thresholding.

Fig. 3(a) shows a sample mammogram image, Fig. 3(b-d) shows the detected clusters using FCM clustering, Otsu thresholding and Active contour models respectively.

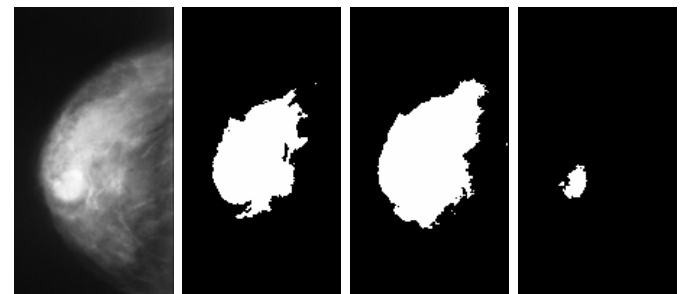


Fig 3a Original Fig 3b FCM Fig 3c Otsu Fig 3d ACM

Fig. 3a indicates a sample mammogram image, Fig3b the detected clusters using FCM clustering, Fig3c Otsu thresholding and Fig 4d Active contour model (ACM)

Visual observation indicates the detection performed by active contour model points more accurately towards the lesion compared with the segmentation results of FCM and Otsu methods. Segmentation error tabulation also supports active contour based segmentation due to the very low segmentation error compared with the other two methods.

Table 1 The segmentation error of these methods

Number of pixels in the image	Error (%)		
	Figure (3b)	Figure (3c)	Figure (3d)
4096	7.81	11.91	1.51

Table 1 Shows that the error due to segmentation of the region of interest is less with proposed method whereas it reaches its maximum value of 11.91 for Otsu thresholding method.

B. Discussion

The smart probe technology [17] incorporates hybrid soft computing algorithms that can help the smart probe to recognize cancerous tissue and help physicians to make better diagnosis assisted by the smart probe technology. Figures 4-6 offer graphical representations of the breast cancer smart probe, 3D visualization interface and smart probe test bed.

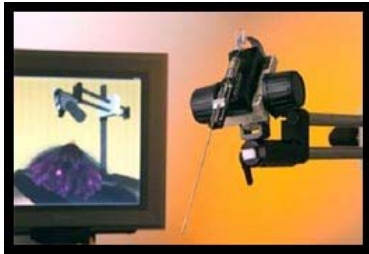


Fig. 4. Breast Cancer Smart Probe (Courtesy: <http://ti.arc.nasa.gov/projects/ssrl/Images/BCDimage1.jpg>)

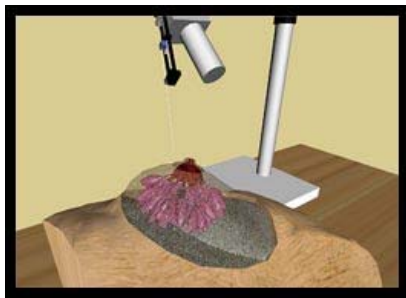


Fig. 5 3D visualization interface (Courtesy: <http://ti.arc.nasa.gov/projects/ssrl/Images/BCDimage2.jpg>)



Fig. 6 Smart probe test bed. (Courtesy: <http://ti.arc.nasa.gov/projects/ssrl/Images/BCDimage3.jpg>)

Judging by the good segmentation results obtained using wavelet based active contour models for mammogram images [18], this paper proposes the incorporation of wavelet based active contour models

for smart probe or similar technology to aid in better isolation and identification of cancerous tissue. Extension of such technique to biopsies can help in minimally invasive and accurate biopsies in breast cancer treatment.

V. CONCLUSION

This research work has proposed a novel smart probe technique for breast cancer treatment using wavelet based active contour model. Better segmentation results yielded by wavelet based active contour models can help to achieve better detection of cancerous tissue by smart probe or similar technology. Extension of this technique to biopsies can help to achieve minimally invasive and accurate biopsies in breast cancer treatment.

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